MAR 428

Test II

Solved Problems

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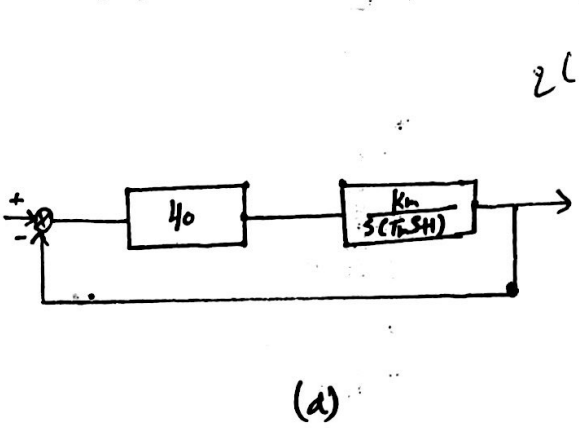
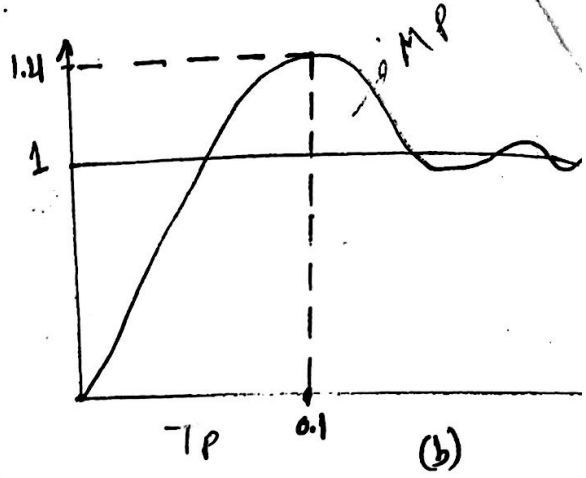
(21) Find the value of K_m and T_m



(a)



(b)

نضع النظام فاصورة دالة النقل التاملة :

$$\frac{40K_m}{T_m S^2 + S + 40K_m} = \frac{40K_m/T_m}{S^2 + \frac{1}{T_m}S + 40K_m/T_m}$$

معامل S لازم يكون 1

$$S^2 + \frac{1}{T_m}S + \frac{40K_m}{T_m} \equiv S^2 + 2\omega_n \zeta S + \omega_n^2$$

من الرسم :

$$M_p = 0.4 = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$\zeta = 0.279$$

$$t_p = 0.1 = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$\omega_n = 32.715 \ \frac{rad}{sec}$$

$$\frac{1}{T_m} = 2\omega_n\zeta$$

$$T_m = \frac{1}{2\omega_n\zeta} = 0.0548$$

$$\frac{40K_m}{T_m} = \omega_n^2$$

$$K_m = \frac{\omega_n^2 T_m}{40} = 1.466$$

1

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(22) For a given transfer function below find ($\zeta, \omega_n, M_p, T_s$)?

$$\frac{C(s)}{R(s)} = \frac{36}{S^2 + 4.2S + 36}$$

$$S^2 + 4.2S + 36 \equiv S^2 + 2\zeta\omega_n S + \omega_n^2$$

$$\omega_n^2 = 36 \qquad \omega_n = 6 \text{ rad/sec}$$

$$2\zeta\omega_n = 4.2 \qquad \zeta = \frac{4.2}{2*6} = 0.35$$

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} = 0.309$$

$$M_{p\%} = 30.9\%$$

$$T_{s2\%} = \frac{4}{\zeta\omega_n} = 1.904 \text{ sec}$$

$$T_{s5\%} = \frac{3}{\zeta\omega_n} = 1.428 \text{ sec}$$

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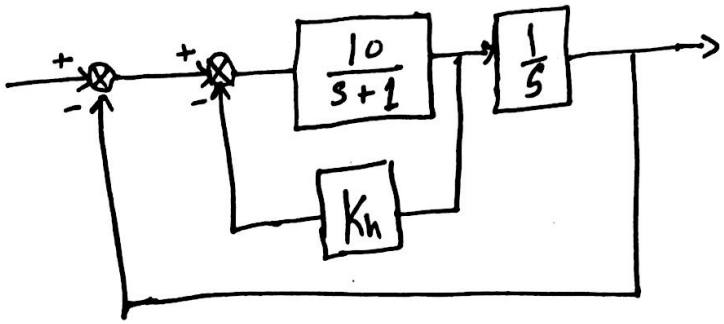
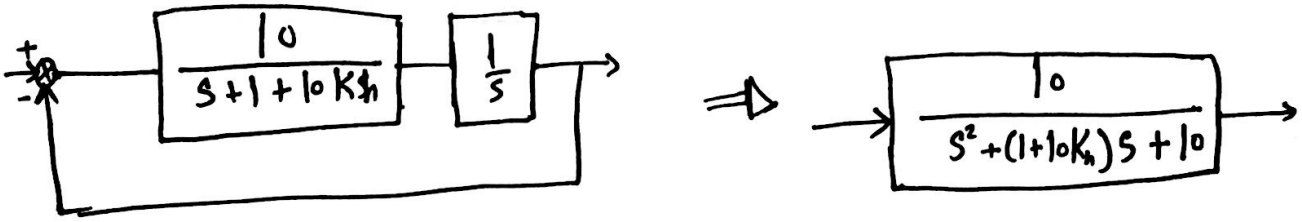
(23) Determine the value of K_h so that the damping ratio ($\zeta = 0.5$)

احسب قيمة K_h التي تجعل $(\zeta = 0.5)$ ؟



١- نضع النظام في صورة دالة النقل الشاملة



$$s^2 + (1+10K_h)s + 10 \equiv s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\omega_n = \sqrt{10} \quad rad/sec$$

$$2\zeta\omega_n = 1 + 10K_h$$

$$K_h = \frac{2\zeta\omega_n - 1}{10}$$

$$K_h = \frac{2*0.5*\sqrt{10} - 1}{10} = 0.21622$$

3

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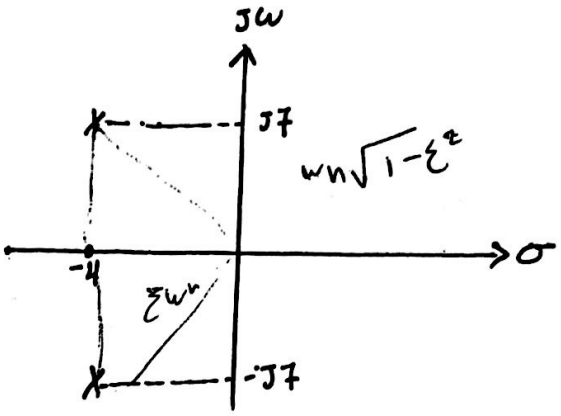
(24) find the transfer function for:



(x) تمثل أقطاب النظام من الدرجة الثانية

$$S = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$S = -4 \pm j7$$

~~GOAL~~

$$\omega_n\sqrt{1-\zeta^2} = 7 \quad (1)$$

$$\zeta\omega_n = 4 \quad (2)$$

$$\omega_n = \frac{4}{\zeta}$$

$$\frac{4}{\zeta}\sqrt{1-\zeta^2} = 7$$

$$\zeta = 0.496$$

$$\omega_n = \frac{4}{0.496} = 8.06$$

الصورة العامة لدالة النظام من الدرجة الثانية:

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$T(s) = \frac{64}{s^2 + 8s + 64}$$

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(27) if a unity negative feedback control system has the following closed Loop transfer function

$$\frac{C(s)}{R(s)} = \frac{Ks+b}{s^2+as+b}$$

فيه خطأ في الطباعة
$Ks+b \Rightarrow as+b$

Determine the open-loop transfer function G(s) and then find the steady state error

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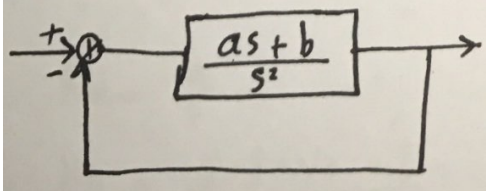
* دالة النقل الشاملة معطاة
* التغذية العكسية $H(s) = 1$ (unity negative feedback)
* أوجد $G(s)$ و e_{ss}

$$\frac{G}{1+GH} = \frac{as+b}{s^2+as+b}$$

~~$1+GH = s^2+as+b$~~

$$G = \frac{as+b}{s^2}$$



النظام من (type 2) إذا الخطأ سيكون له قيمة في حالة (~~Step input~~ parabolic input) فقط

$$e_{ss} = \frac{1}{K_a}$$

$$K_p = \lim_{s\to 0} s^2 G(s) = s^2 \frac{as+b}{s^2} = b$$

$$e_{ss} = \frac{1}{b}$$

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(28) The forward transfer function of an control system has three poles at 1, 2, 3, what is the system type?

دالة المسار الأمامي لنظام [G(s)] تمتلك الأقطاب التالية 1, 2, 3
أوجد نوع النظام ؟

$$G(s) = \frac{1}{(s-1)(s-2)(s-3)}$$

$$G(s) = \frac{1}{(s-1)(s^2-5s+6)}$$

$$G(s) = \frac{1}{s^3-5s^2+6s-s^2+5s-6}$$

$$G(s) = \frac{1}{s^3-6s^2+11s-6}$$

* النظام من النوع 0
* رتبة النظام 3
* المعادلة المميزة $C.E \Rightarrow s^3-6s^2+11s-6=0$

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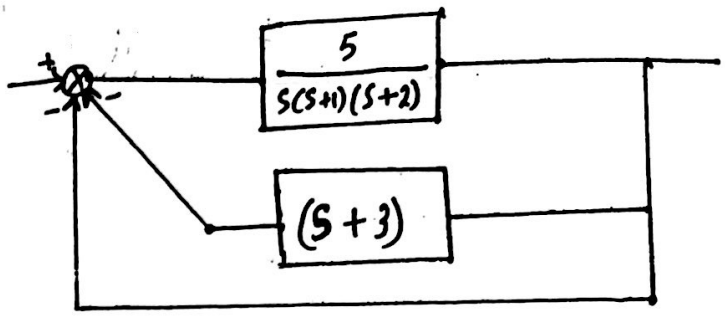
(29) For the system shown in the following figure

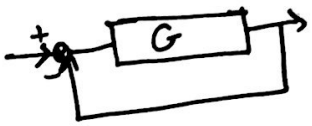
① Find K_p, K_v, K_a

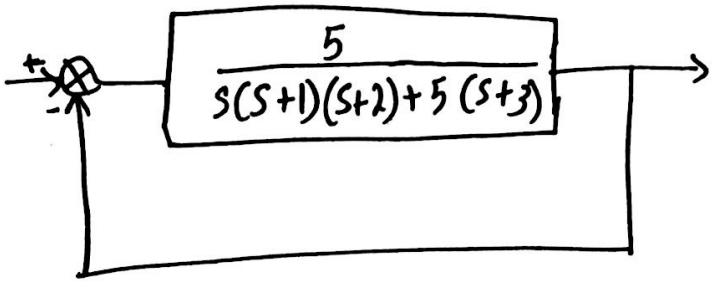
② e_{ss} for $50u(t), 50tu(t), 50t^2u(t)$

③ system type



١- نضع النظام في صورة :





$$\frac{G \text{ بسط}}{G \text{ مقام} + (H) \text{ في } (G) \text{ بسط}}$$

$$K_p = \lim_{s\to 0} G(s) = \frac{5}{0+5(3)} = \frac{1}{3}$$

$$K_v = \lim_{s\to 0} sG(s) = s\ \frac{5}{0+5(3)} = 0$$

$$K_a = \lim_{s\to 0} s^2 G(s) = s^2\ \frac{5}{0+5(3)} = 0$$

نوع النظام (type 0)

step input $R(s) = 50/s$

$$e_{ss} = \frac{50}{1+K_p} = \frac{50}{1+\frac{1}{3}}$$

Ramp input $R(s) = 50/s^2$

$$e_{ss} = \frac{50}{K_v} = \frac{50}{0} = \infty$$

parabolic input $R(s) = 50/s^3$

$$e_{ss} = \frac{50}{K_a} = \frac{50}{0} = \infty$$

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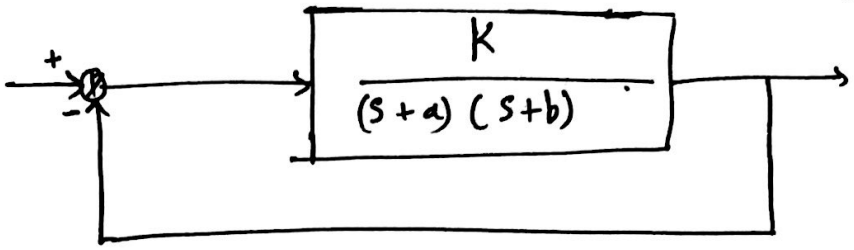
(32) Find the sensitivity of steady-state error to change to K and b ?



$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\lim_{s\to 0} G(s)} = \frac{1}{1+\frac{K}{ab}} = \frac{1}{\frac{ab+K}{ab}} = \frac{ab}{ab+K}$$

$$S_K^e = \frac{\partial e}{\partial K} \cdot \frac{K}{e} = \frac{(ab+K)(0) - ab(1)}{(ab+K)^2} * \frac{K}{\frac{ab}{(ab+K)}} = \frac{-abK}{(ab+K)ab}$$

$$S_K^e = \frac{-K}{(ab+K)}$$

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(4o) Determine the characteristic equation for the follo unity negative feedback system define by its open-loop transfer function $G(s) = \frac{K}{(s+49)(s^2+4s+5)}$

and find ⓐ K for stability

ⓑ K for marginal stability

ⓒ the actual location of the poles when the system is marginally stable

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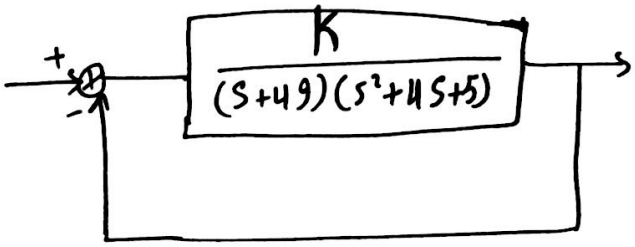
يجب وضع النظام في صورة دالة النقل الشاملة حتى نوجد C.E !

$$= \frac{K}{(s+49)(s^2+4s+5)+K}$$



$$T(s) = \frac{K}{s^3+4s^2+5s+49s^2+196s+245+K}$$

$$T(s) = \frac{K}{s^3+53s^2+201s+(245+K)}$$

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C.E $S^3 + 53S^2 + 201S + (245+K)$

S^3	1	201	0
S^2	53	$(245+K)$	0
S^1	$\frac{10408-K}{53}$	0	
S^0	$245+K$		

$$b_1 = -\frac{\begin{vmatrix} 1 & 201 \\ 53 & 245+K \end{vmatrix}}{53} = -\frac{(-10408+K)}{53}$$

$$b_2 = -\frac{\begin{vmatrix} 1 & 0 \\ 53 & 0 \end{vmatrix}}{53} = 0$$

$$C_1 = -\frac{\begin{vmatrix} 53 & (245+K) \\ \frac{10408-K}{53} & 0 \end{vmatrix}}{\frac{10408-K}{53}} = 245+K$$

لكي يكون النظام مستقر:

$10408 - K > 0$

$K < 10408$

$245 + K > 0$

$K > -245$

$-245 < K < 10408$

لكي يكون النظام مستقر استقرار حرج

$K = 10408$

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